

Ch 4 p1

MA121

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Exponential and Logarithmic Functions

↳ 4.1 exponential growth & decay.

Let us say we deposit an initial amount of money, V_0 , in a bank. It earns us interest at an effective rate, r . After one year, the money will have grown from V_0 to V_1 . The effective rate of growth per year is defined as

$$r = \frac{V_1 - V_0}{V_0} = \frac{V_1}{V_0} - \frac{V_0}{V_0} = \boxed{r_{\text{eff}} = \frac{V_1}{V_0} - 1}$$

Ex 2P105

The average value of a 1200 ft² Condo in downtown Saskatoon at a certain year was \$500,000. The next year, the average value was \$525,000. What was the effective rate of growth after one year?

given $V_0 = \$500,000$, and $V_1 = \$525,000$

$$\therefore r_{\text{eff}} = \frac{V_1}{V_0} - 1$$
$$= \frac{525000}{500000} - 1$$

$$= 1.05 - 1$$

$$= 0.05, \text{ or } 5\%.$$

NOTE

↳ if $V_1 > V_0$, the value is said to be growing.
we have $r_{\text{eff}} > 0$

↳ If $V_1 < V_0$, the value is said to be decaying.
we have $r_{\text{eff}} < 0$

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Ex 3P105

During the span of one year, the Blue Book value of a certain truck declined at an effective rate of 4.5% to the value of \$27,000. What was the value of the truck the previous year? given the truck's value decline at an effective rate of 4.5%. so, $r_{\text{eff}} = -4.5\%$ ← because of declining value.

$$\Rightarrow r_{\text{eff}} = -0.045$$

$$\text{so, } r_{\text{eff}} = \frac{V_1}{V_0} - 1, \text{ given } V_1 = \$27000$$

$$\text{Thus, } -0.045 = \frac{27,000}{V_0} - 1$$

$$\Rightarrow \frac{27,000}{V_0} = -0.045 + 1$$

$$\Rightarrow \frac{27,000}{V_0} = 0.955$$

$$\Rightarrow \frac{27,000}{0.955} = V_0$$

$$\Rightarrow V_0 \approx \$28,272.25$$

Suppose $V(t_0)$ is the initial value. The effective rate of growth is $r_{\text{eff}} = \frac{V_1}{V_0} - 1$

$$\Rightarrow r_{\text{eff}} = \frac{V(t_0+1)}{V(t_0)} - 1, \text{ where } V(t_0) = \text{year we start investing.}$$

$$\Rightarrow \frac{V(t_0+1)}{V(t_0)} = 1 + r_{\text{eff}}$$

$$\Rightarrow V(t_0+1) = V(t_0)(1 + r_{\text{eff}}) \leftarrow \text{value after one year.}$$

If the effective rate of growth or decay does not change, and we do not touch our investment, then the value after two years will be

$$r_{\text{eff}} = \frac{V(t_0+2)}{V(t_0+1)} - 1$$

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$$\Rightarrow V(t_0 + 2) = V(t_0 + 1)(1 + r_{\text{eff}})$$

$$\Rightarrow V(t_0 + 2) = V(t_0)(1 + r_{\text{eff}})(1 + r_{\text{eff}})$$

$$\Rightarrow V(t_0 + 2) = V(t_0)(1 + r_{\text{eff}})^2 \leftarrow \text{value after 2 years}$$

If nothing changes, and we let our investment ride, $r_{\text{eff}} = \frac{V(t_0 + 3)}{V(t_0 + 2)} - 1$

$$\Rightarrow V(t_0 + 3) = V(t_0 + 2)(1 + r_{\text{eff}})$$

$$\Rightarrow V(t_0 + 3) = V(t_0)(1 + r_{\text{eff}})^2(1 + r_{\text{eff}})$$

$$\Rightarrow V(t_0 + 3) = V(t_0)(1 + r_{\text{eff}})^3 \leftarrow \text{value after 3 years}$$

If nothing changes, we continue to let the investment ride on. After t years, our investment grows/decays to

$$V(t_0 + t) = V(t_0)(1 + r_{\text{eff}})^t$$

DEMO

$V(t_0) = \$100$ Subject to $r_{\text{eff}} = 5\% = 0.05$

After $t = 1$ year,

$$V(t_0) + 1 = \$100(1 + 0.05)^1 = \$105.$$

After $t = 2$ years,

$$V(t_0) + 2 = \$100(1 + 0.05)^2 = \$110.25$$

After $t = 3$ years,

$$V(t_0 + 3) = \$100(1 + 0.05)^3 = \$115.76$$

After $t = 20$ years,

$$V(t_0 + 20) = \$100(1 + 0.05)^{20} = \$265.33$$

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rewriting our model....

let P = Principle (initial) value

r_{eff} = Interest rate, t = # of years

and $F(t)$ = Future value after t number of years.

$$F(t) = P(1 + r_{\text{eff}})^t$$

Notice that this formula can be thought of as an exponential form $y = ab^x$

Analogy $\rightarrow y = F(t)$

$a = P$, a is the coefficient

$b = 1 + r_{\text{eff}}$, b is the base

$x = t$, x is the variable

One important condition for an exponential to be useful is $b > 0$ the domain will then be $x \in (-\infty, \infty)$

Two Possibilities:

$\rightarrow r_{\text{eff}} > 0$, we have growth.

and $1 + r_{\text{eff}} > 1$

$\Rightarrow b > 1$

$\rightarrow r_{\text{eff}} < 0$, we have decay.

and $1 + r_{\text{eff}} < 1$

$\Rightarrow b < 1$

But since we want $b > 0$, for decaying situations we have $0 < b < 1$, so if b is between 0 and 1, you have a decaying situation.

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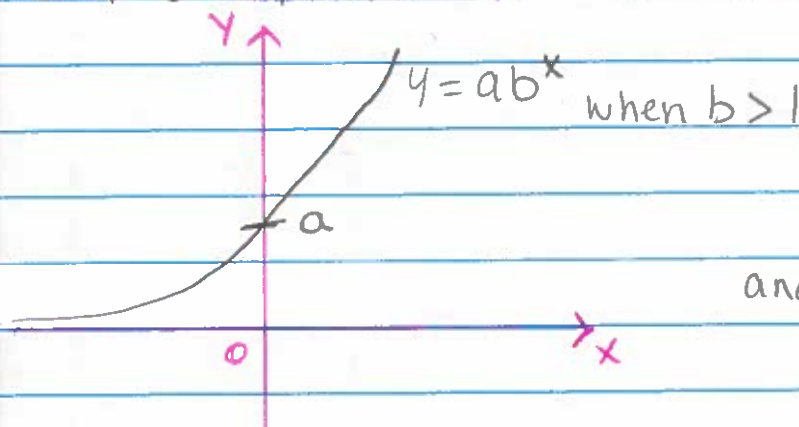
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For all exponential functions, the y-intercept occurs when $x=0$, so $y=ab^0 \Rightarrow y=a$.

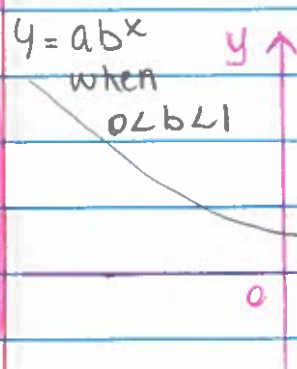
Therefore, the y-intercept occurs at $(0, a)$. Since $a > 0$, and $b > 0$, $y > 0$ for all x .

Graphs of $y=ab^x$



$$\lim_{x \rightarrow -\infty} (ab^x) = 0$$

$$\text{and } \lim_{x \rightarrow \infty} (ab^x) = \infty$$



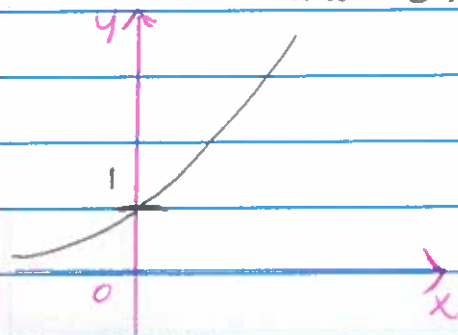
$$\lim_{x \rightarrow -\infty} (ab^x) = \infty$$

$$\text{and } \lim_{x \rightarrow \infty} (ab^x) = 0$$

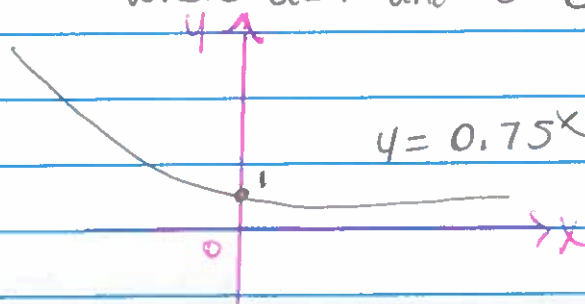
Ex

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i) $y=x^3 \Rightarrow y=1(3^x)$
where $a=1$ and $b>1$



ccc) $y=-0.75^x$
lets draw $y=1(0.75^x)$
where $a=1$ and $0 < b < 1$

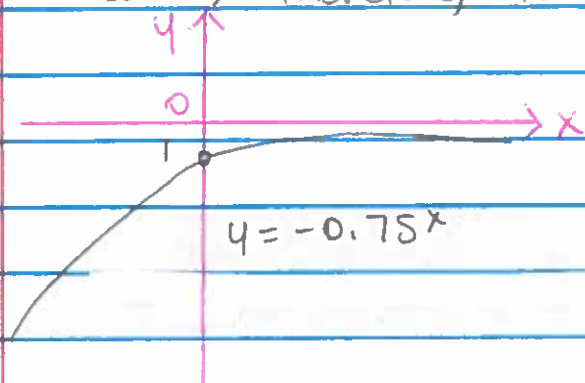


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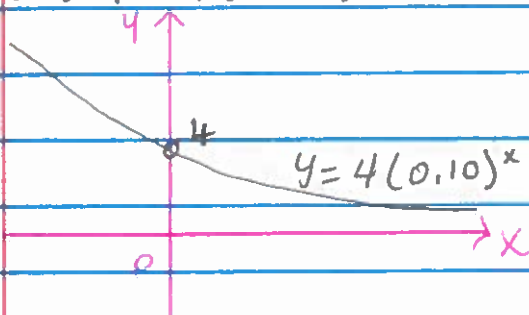
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iii) can't) therefore, the graph of $y = -0.75^x$ is



iv) $y = 4(0.10)^x$ where $a = 4$ and $0 < b < 1$



ex.

6 p 106

given $r_{\text{eff}} = 2.25\% = 0.0225$ and $t = 18$ years

We want $F(18) = \$10,000$. What is P ?

$$F(t) = P(1 + r_{\text{eff}})^t \quad \leftarrow \text{plug in values...}$$

$$\$10,000 = P(1 + 0.0225)^{18}$$

$$\Rightarrow P = \frac{10,000}{(1.0225)^{18}}$$

$$\Rightarrow P \approx \$6,699.78$$

ex.

8 p 106

during the first 3 years,

Let $P = \$80$, $r_{\text{eff}} = 4\% = 0.04$, and $t = 3$ years.

$$F(t) = P(1 + r_{\text{eff}})^t$$

$$F(3) = 80(1 + 0.04)^3$$

$$\Rightarrow F(3) \approx 589.99$$

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8 cont'd) The next 3 years,

$P = 89.99$, $r_{eff} = 0.04$, $t = 3$ years.

$$F_2(3) = 89.99 (1 - 0.04)^3$$

$$F_2(3) \approx \$79.62$$

Something to think about.

What happens if Anastasia's situation is 4% decay for 3 years followed by 4% growth for the next 3 years?